PHILOSOPHY AND NATURE OF MATHEMATICS

Voice of Research
Volume 4, Issue 1
June 2015
ISSN 2277-7733

Amarnath Murthy
DGM (E&T), RO Infocom, ONGC, Mumbai

Abstract

There is nothing in our lives, in our world, in our universe, that cannot be expressed with mathematical theories, numbers, and formulae. Mathematics is the queen of science and the king of arts; to me it is the backbone of all systems of knowledge. Mathematics is a tool that has been used by man forages. It is a key that can unlock many doors and show the way to different logical answers to seemingly impossible problems. Not only can it solve equations and problems in everyday life, but it can also express quantities and values precisely with no question or room for other interpretation. There is no room for subjectivity. Though there is a lot of mathematics in politics, there is no room for politics in mathematics. Coming from a powerful leader two + two can not become five it will remain four. Mathematics is not fundamentally empirical —it does not rely on sensory observation or instrumental measurement to determine what is true. Indeed, mathematical objects themselves cannot be observed at all! Mathematics is a logical science, cleanly structured, and well-founded.

Keywords: *Mathematics, philosophy, number, vedic mathematics*

Richard Feynman. Says: "To those who do not know Mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty of nature. ... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in."

The Role of Mathematics: While there are exceptions, it is generally true that great mathematics is studied for its own sake and without reference to anything outside itself. Mathematics has a beauty all its own and there is, for the mathematician, an aesthetic joy that comes from solving an important problem, no matter what value society may place on this activity. In this sense, mathematics has constantly sought to free itself from its practical origins. Geometry, for example, began with rules for surveying, calculating the areas of fields and making astronomical studies and acts of navigation. Probability theory had its origins in the desire to rise gambling to a high art. But, very quickly, mathematics shook itself free from such pedestrian origins. While it is certainly true that some exceptional mathematicians have begun their studies with a concrete problem taken from the physical world, in the end, the mathematics they have developed has moved away from these specific cases in order to focus on more abstract relationships. Mathematics is not really concerned with specific cases but with the abstract relationships of thought that spring from these particular instances. Indeed, mathematics takes a further step of abstraction by investigating the relations between these relationships. In this fashion, the whole field moves away from its historical origins, towards greater abstraction and increasing beauty.

Power and beauty of Mathematics: Where we find certainty and truth in mathematics we also find beauty. Great mathematics is characterized by its aesthetics. Mathematicians delight in the elegance, economy of means, and logical inevitability of proof. It is as if the great mathematical truths just have to be that way and no other. This light of logic is also reflected back to us in the underlying structures of the physical world through the mathematics of theoretical physics. Mathematics is the part of science you could continue to do if you woke up tomorrow and discovered the universe was gone. Galileo said: "We cannot understand [nature] if we do not first learn the language and grasp the symbols in which it is written." Mathematics is truly a common language that will always express the same quantity, event, or position. Focus on expressing the same quantity in theory; in our world, two plus two equals four. This is because "twoness" and "fourness" are real entities that hold up. In another world where atoms were the size of houses, people walked on their hands instead of their feet, and galaxies were not shaped like spirals but like rectangles and squares, two plus two would still be four. Though physical properties may differ in different worlds, mathematics still holds true. Mathematicians are mad tailors: they are making "all the possible clothes" hoping to make also something suitable for dressing... The dream of structuring the world according to mathematical principles began long before the rise of modern science. No theory however abstract in mathematics has ever gone unutilized or wasteful. Theories which were developed by pure mathematicians just for their own delight and aesthetic joy with no relation to any physical/ world problem then, have also found astonishing applications unimaginable by the discoverer. In 1864 James Clerk Maxwell proved mathematically through his equations the existence of electromagnetic waves. And based on that in 1887 Heinrich Hertz sent and received wireless waves, using a spark transmitter and a resonator receiver. *One surprising example of the utility of mathematics is, even the difficulty of being able to solve a problem i.e. prime factorization has the most modern application in 'cryptography' the science of using mathematics in encryption and decryption of coded messages in military intelligence and computer security. One can, in a way pass it (the difficulty involved in solving a problem) on to the enemy. The military intelligence power of a nation is rated and based on strength and availability of this science i.e. the largest prime number known.* The NSA National Security Agency of USA is said to be the largest single employer of mathematicians in the world. The NSA is estimated to have about 40,000, employees. *The acronym NSA is aptly said to stand for "Never Say Anything."* There is a saying that*" these days the purest of the pure mathematics is in danger of being applied. Relations between pure and applied mathematicians are based on trust and understanding. Namely, pure mathematicians do not trust applied mathematicians, and applied mathematicians do not understand pure mathematicians.*

The most elegant equation in mathematics: The seven most important symbols in mathematics are e, i, π, 1, 0, + and = Using these symbols each only once, Euler had derived a beautiful equation which in my opinion is the most powerful and elegant result in mathematics. $e^{i} + 1 = 0$ Pythagoras theorem of course is the oldest most useful and beautiful result in mathematics.

History of mathematics: Babylonian and Egyptian mathematics emphasized arithmetic and the idea of explicit calculation. But Greek mathematics tends to focus on geometry and increasingly relied on getting results by formal deduction. For being unable to draw geometrical figures with infinite accuracy this seemed the only way to establish anything with certainty. And when Euclid around 330 BC did his work on geometry he started from 10 axioms and derived 465 theorems. Euclid's work was widely studied for more than two millennia and viewed as a quintessential example of deductive thinking. But in arithmetic and algebra which in effect dealt mostly with discrete entities, a largely calculational approach was still used. In 1600s and 1700s, however, the development of calculus and notions of continuous functions made use of more deductive methods. Often the basic concepts were somewhat vague, and by mid 1800s, as mathematics became more elaborate and abstract, it became clear that to get systematically correct results a more rigid structure would be needed. The introduction of non-Euclidian geometry in 1820s, followed by various forms of abstract algebra in the mid 1800s, and transfinite numbers in 1880s indicated that mathematics could be done with abstract structures that have no obvious connections to everyday intuition. Set theory and predicate logic were proposed as ultimate foundations for all of mathematics. But at the very end of 1800s paradoxes were discovered in these approaches. And there followed an increasing effort- notably by David Hilbert to show that everything in mathematics could consistently be derived just by starting from axioms and using formal processes of proof. Gödel's theorem showed in 1931 that at some level this approach was flawed. In any system rich enough to support the axioms of arithmetic, there will exist statements that bear a truth value, but can never be proved or disproved. Mathematics cannot prove its own consistency. But by 1930s pure mathematics had already firmly defined itself to be on the notions of doing proofs and indeed for the most part continues to do so even today. In recent years, however the increasing application of explicit computation has made proof less important, at least in most applications of mathematics.

Vedic mathematics: The Vedas are the most ancient record of human experience and knowledge,passed down orally for generations and written down about 5,000 years ago. Medicine, architecture, astronomy and many other branches of knowledge, including maths, are dealt with in the texts. Vedic Mathematics is the name given to the ancient system of Mathematics which was rediscovered from the Vedas between 1911 and 1918 by Sri Bharati Krsna Tirthaji (1884-1960). According to his research all of mathematics is based on sixteen Sutras or word-formulae. For example, 'Vertically and crosswise` is one of these Sutras. These formulae describe the way the mind naturally works and are therefore a great help in directing the student to the appropriate method of solution. The simplicity of Vedic Mathematics means that calculations can be carried out mentally (though the methods can also be written down). There are many advantages in using a flexible, mental system. Pupils can invent their own methods; they are not limited to the one 'correct' method. This leads to more creative, interested and intelligent pupils. But the real beauty and effectiveness of Vedic Mathematics cannot be fully appreciated without actually practising the system the whole approach of Vedic maths is suitable for slow learners, as it is so simple and easy to use.

Mathematical ideas: Mathematics is not just about calculations but about ideas. Not all ideas are mathematics but all good mathematics must contain an idea. Five distinct sources of mathematical ideas are Number, Shape, Arrangement, Movement and Chance

Number: Originally the number concept must have arisen through counting: possessions, days, enemies. Measurement of lengths and weights led to fractions and the real numbers. With the creation of imaginary numbers mathematics was never quite the same.

Shape: Shape or form leads to geometry (Euclidian) and the modern offspring such as topology, singularity theory, Lie groups and gauge field theory. Novel geometric forms: fractals, catastrophe, fibre bundles, strange attractors etc.

Arrangement: Ways to arrange objects according to various rules lead to combinatorics, parts of modern algebra and number theory and what is known as "finite mathematics" the basis of much of computer science.

Movement: Of Cannon balls, planets, or waves inspired calculus, theory of ordinary and partial differential equations, calculus of variations, and topological dynamics. Many of the biggest area of mathematical research concern the way system evolve in time.

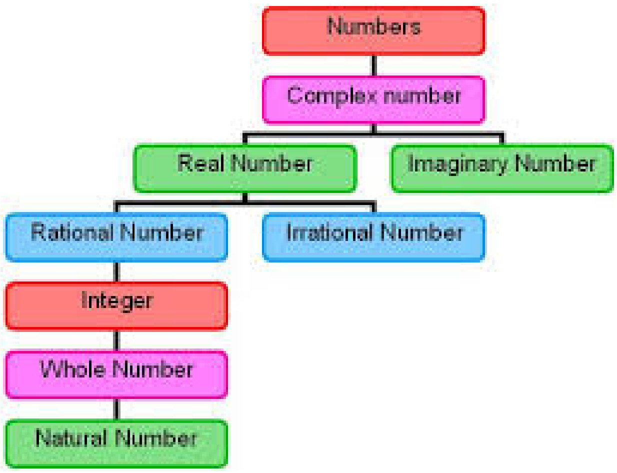
Chance: A more recent ingredient is chance or randomness, (Probability and statistics). Only for a couple of centuries has it been realised that chance has its own type of pattern and regularity (Fractals).

Classification of mathematics

MATHEMATICS			
DESCRETE		CONTINUOUS	
ABSOLUE	RELATIVE	STATIC	DYNAMIC
ARITHMETIC	MUSIC	GEOMETRY	ASTRONOMY

What is Number? We use numbers every day and tend to take them for granted. But how did the idea of numbers arise? Is counting intuitive or did it arise to solve particular problems? Some of the oldest evidence of counting so far discovered comes from ancient artifacts belonging to groups of hunters and gatherers. For example, a wolf bone, dated about 30,000 BC, has been discovered with a series of notches carved in it, which seem to represent a tally of some kind. Tallying seems to be the earliest methods of keeping a record of quantities and appears in many cultures. But is this really counting? For example, a tally system might be used to keep track of a flock of sheep. A small stone might be put in a pile for each sheep as it is let out to graze in the morning and then a stone could be removed from the pile again for each sheep as it was collected in at night. Any pebbles left over would indicate that some sheep were missing. This is really only a straight comparison between two sets of objects, the stones and the sheep. No idea of the actual number of sheep in the flock. This is counting through mapping (one to one correspondence). At some developmental stage of a culture different number words were being used depending on the context e.g. there might be one word for four people and another for four stones. At some stage an abstract idea of number develops and the concept of, "threeness", where a group of three fish and three stones are perceived to have something in common, is incorporated into the system. One, Two, Many Systems, There were cultures that had number systems consisting only of words for one, two, and many with words for different types of "many" only being developed later.

Types of numbers: 1).*Cardinal numbers:* how many objects? 5, 7… 2). *Ordinal numbers:* Order (position) of a element in a set.(3^{rd}, 4^{th}) 3). *Tag numbers:* Gives a unique name to an object: bus number, telephone number etc. The number concept evolved as, natural numbers, zero, integers (positive and negative), rational numbers (½, 3/5...), irrational numbers which included algebraic numbers ($2^{1/2}$, $3^{1/2}$...) and transcendental numbers(π, e...), and finally imaginary numbers/complex numbers. Complex numbers are like God!!! Here is the analogy. Can you imagine that there is a task which you and I are capable of doing but God is not. Surprising!!! What is that? In case I am angry with someone I can ask him to get out of my house but can God do it? God cannot. Because God cannot disown his creatures and also there is no limit to its territory. Set of rational numbers disowns π and $2^{1/2}$, real numbers can disown imaginary numbers but complex numbers like God cannot disown any number. Every number can be written as (a + bi) which means every number is a complex number.



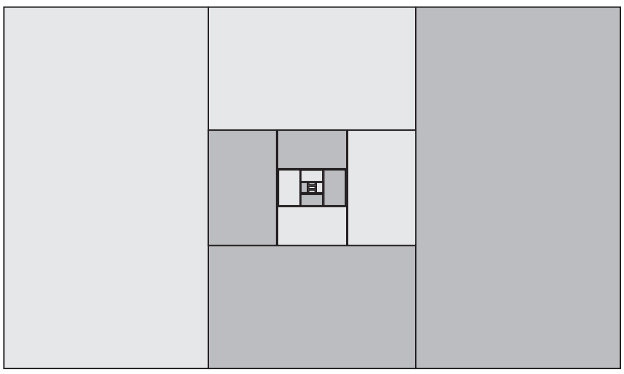
Cantor shocked the world by showing that the real numbers are not countable... there are "more" of them than the integers! In a way he proved that there are infinitely many kinds of infinities.

Some terminology: A theorem is a statement that can be shown to be true (via a proof). A proof is a sequence of statements that form an argument. Axioms or postulates are statements taken to be self evident or assumed to be true. A lemma is a theorem useful within the proof of a theorem. A corollary is a theorem that can be established from theorem that has just been proven. A proposition is usually a 'less' important theorem. A conjecture is a statement whose truth value is unknown.

The idea of proof in mathematics: Mathematicians (and scientists to a large extent) live in a world in which some things are certain. Mathematicians have Euclid's (and other people's) axioms, postulates, and theorems. Physicists have Newton's (and other people's) laws. These are ideas that are so basic that it would be silly to deny them (under normal circumstances). And mathematicians and scientists use these postulates, theorems, and laws to deduce other theorems and laws. This is proof. Given these postulates then this is a result. (if – then). "Mathematical proofs, like diamonds, are hard and clear, and will be touched with nothing but strict reasoning." *(John Locke).* Mathematical proofs are, in a sense, the only true knowledge we have. They provide us with a guarantee as well as an explanation (and hopefully some insight).

Various types of proofs: *Existence proof:* Usually, to show that some things with certain properties exist, just show an example. To prove there is an even prime number, just mention 2. If an example cannot be found, this can be a very difficult kind of proof indeed. *Proof by contraposition:* If we have a statement of the form A implies B (If A then B), then the contrapositive is: not B implies not A. A statement is always equivalent to its contrapositive. If you have proved one, you have proved the other. *Backward proof:* Assume the intended theorem is true. See that it leads to basic truths. This is a flawed method; it is not a proof. It may help one discover a proof, however. It is necessary to see if one can make the backward proof go forward. The backward proof is sometimes diabolical. At worst, you can assume something that is false and end up with some obvious truth, and you think you proved your false statement. At best, you assume something that is true, and end up mistakenly thinking you have proved it. The following erroneous proof is too obvious to fool anyone: Assume 3=2. Then by the commutative law, 2=3. Add the two equations, and we get 5=5. Subtract 5 from both sides, and we get 0=0, which I happen to know is true. Therefore 3=2. Not! To actually try to prove that 3=2, we would have to start with 0=0 and end with 3=2. And that won't work. *Proof by Contradiction:* To prove a statement p is true you may assume that it is false and then proceed to show that such an assumption leads a contradiction with a known result. Examples (1) Pythagoras' proof that the square root 2 is not a rational number. (2) Euclid's proof that there are infinitely many primes.

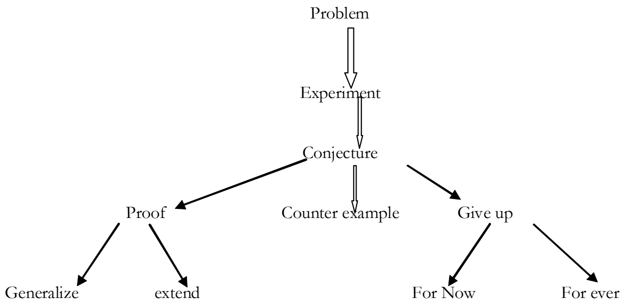
Proof by mathematical induction: Guess at a formula, and then prove it. This proof is stated informally. Example: $(1+2+...+n)=n(n+1)/2$. *Proof by infinite decent :* Fermat proved that sum of two integer cubes cannot yield a third cube by proving that if such an example exists then there exists yet another case with smaller numbers and so on. *Impossibility proof:* (1) trisection of an angle using ruler and compass. (2) Squaring a circle. (3) Doubling a cube etc. *Proof without words:* $1/3 + 1/9 + 1/27 + \dots + 1/3^n = ½$



1/3 1/9 1/3

The pink area = The blue area = ½ = $1/3 + 1/9 + 1/27 + \dots + 1/3^n$ *Proof by cases:* Sometimes it is easier to prove a theorem by breaking it down into cases and proving each one separately. *Proof by exhaustion:* Proofs which analyse and document every instance of an assertion in a case-wise analysis can be used in finite cases. This is a cumbersome method of proof and is really only suitable when dealing with fairly small sets. A more involved use of proof by exhaustion is the proof of the Four-Colour Theorem. The proof has broken down the four-colour problem into subsets and a four-colour proof was applied to each of these by a computer. This proof has not currently been verified by a human and is hence not considered fully proven. No matter how many examples you give, you can never prove a theorem by giving examples (unless the universe of discourse is finite in which case it is called an exhaustive proof). Counter examples can only be used to disprove universally quantified statements.

Flowchart of growth of mathematics



Some funny definitions of a mathematician: 1. A math professor is one who talks in someone else's sleep. 2. A Mathematician is a device for turning coffee into theorems. 3. A blind man looking for a black cat in a dark room which is not there. 4. A statistician can have his head in an oven and his feet in ice, and he will say that on the average he feels fine.

Smartness of a mathematician: One day a farmer called up an engineer, a physicist, and a mathematician and asked them to fence off the largest possible area with the least amount of fence. The engineer made the fence in a circle and proclaimed that he had the most efficient design. The physicist made a long, straight line and proclaimed "We can assume the length is infinite..." and pointed out that fencing off half of the Earth was certainly a more efficient way to do it. The Mathematician just laughed at them. He built a tiny fence around himself and said "I declare myself to be on the outside."

Right approach towards mathematics: The world today is full of complex systems and mathematics is the subject that's supposed to provide models, framework and metaphor for thinking quantitatively about those things. Mathematical skills are essential for children if they are to flourish to their fullest in the fast moving technological world of tomorrow. *Calculation and rule- following only make up a small part of big picture in terms of what students need to know, but because they are the easiest skills to measure, that's what schools emphasize. And children who don't grasp it right away don't enjoy doing it often end up fearful of all mathematics.* Bookish mathematics divorced from life and its everyday problems can be boring and seem meaningless; cripple the enthusiasm and dull the inquiring mind of the young student. But when it is learned as a key to the solution of everyday problems, mathematics can be exciting, give students a sense of power and achievement. The most beautiful thing about mathematics and its study is that you don't require a formal education to start learning it. In fact if your slate is clean you will learn faster and enjoy more. The basic assumptions are very few and with these you can embark upon a journey that is thrilling, absorbing and highly rewarding. And if you continue this process of learning, on your way you will have discovered the real meaning of this life and after life. You can start this journey alone and still not get bored even for a single moment. Recall the great Indian self-taught mathematician Srinivasa Ramanujan. In fact mathematics can be enjoyed as one enjoys music with the only a difference that one can enjoy music without knowing anything about it but to enjoy mathematics some basic knowledge is essential and afterwards there is no looking back. It is also true that trained musicians enjoy music much more than the listeners. And to learn music or mathematics it requires dedication. I do not know if it is true - that when one of the kings was trying to learn geometry from Euclid he complained that it was difficult. And Euclid said, "There is no royal road to geometry". Mathematics gives shortcuts to everything but there is no shortcut to learning mathematics.

Final word: All life is biology. All biology is physiology. All physiology is chemistry. All chemistry is physics. All physics is math."(Marquardt, 2001) Mathematics is obviously the most interesting, entertaining, fascinating, exciting, challenging, amazing, enthralling, thrilling, absorbing, involving, fascinating, mesmerizing, satisfying, fulfilling, inspiring, mindboggling, refreshing, systematic, energizing, satisfying, enriching, engaging, absorbing, soothing, impressive, pleasing, stimulating, engrossing, magical, musical, rhythmic, artistic, beautiful, enjoyable, scintillating, gripping, charming, recreational, elegant, unambiguous, analytical, hierarchical, powerful, rewarding, pure, impeccable, useful, optimizing, precise, objective, consistent, logical, perfect, trustworthy, eternal, universal subject in existence full of eye catching patterns. So many adjectives might irk anybody but truly ... It is independent of time and space. Applied mathematics is the science of patterns and order and the study of measurement, properties, and the relationships of quantities; using numbers and symbols. Mathematics is the key to understanding our world around us. It is perhaps the purest of the pure mental endeavor of humankind. Mathematics is such a beautiful game of numbers, notions and notations designed and created by humankind that even nature behaves mathematically. Mathematics is fun.

References

abinitiodotgre.wordpress.com
A History of Mathematics viny - Academia.edu
F David Peat - *From Certainty to Uncertainty*
homepages.ihug.co.nz/~sai/peat_uncertainty.htm.
http://en.wikibooks.org/wiki/Mathematical_Proof/Methods_of_Proof/Other_Proof_Types
Ian Stewart *Problems of Mathematics*
NUMBER SYSTEM [Theory 01] | Ab-initio-gre
www.academia.edu/6136577/A_History_of_Mathematics_viny